

Part 1, MULTIPLE CHOICE, 5 Points Each

1 In a survey of 10 students, each was asked to count the number of keys they were carrying. The results were as follows:

1, 1, 1, 2, 2, 2, 2, 2, 3, 5.

Calculate the average number of keys carried by the students surveyed.

- (a) 2 (b) 2.5 (c) 3 (d) 2.1 (e) 21

2 A random variable X has the following probability distribution:

| k | $\Pr(X = k)$ |
|-----|--------------|
| -1 | $1/2$ |
| 0 | $1/4$ |
| 1 | $1/8$ |
| 2 | $1/8$ |

Find the Expected value of the random variable X .

- (a) 1 (b) $\frac{-1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$ (e) 0

3 Mr. Drillit, the dentist of Tooth Acres, has determined from his records the probability distribution of the variable $X =$ the number of fillings he administers in a day. The probability distribution for the random variable X is given below. Find the variance, σ^2 , of X .

| k | Pr($X = k$) |
|---|---------------|
| 2 | $\frac{1}{8}$ |
| 3 | $\frac{1}{8}$ |
| 4 | $\frac{1}{2}$ |
| 5 | $\frac{1}{8}$ |
| 6 | $\frac{1}{8}$ |

- (a) 0 (b) $\frac{5}{4}$ (c) $\sqrt{\frac{5}{4}}$ (d) $\frac{7}{4}$ (e) $\sqrt{\frac{7}{4}}$

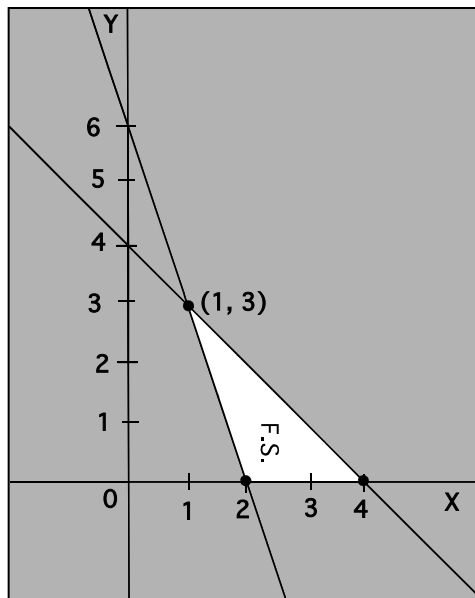
4 Find the area under the standard normal curve between $z = -1.5$ and $z = 3.1$.

- (a) 0.5 (b) 0.9990 (c) 0.0658 (d) 0.0010 (e) 0.9332

5 The amount of milk contained in a gallon container is normally distributed with mean 128.2 ounces and standard deviation 0.2 ounces. What is the probability that a random bottle contains less than 128 ounces?

- (a) 0.3085 (b) 0.8413 (c) 0.1587 (d) 0.6915 (e) 0.5

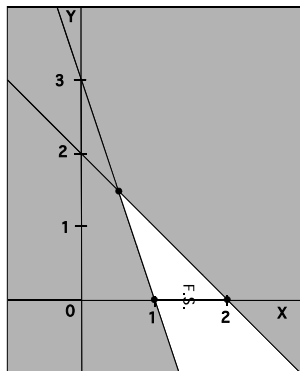
6 What is the maximum of the objective function $4x + y$ on the feasible set in the following diagram?



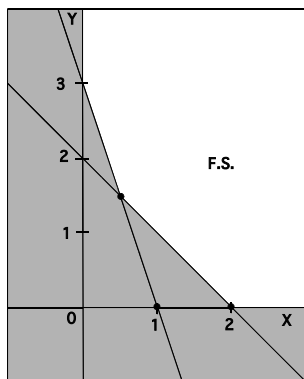
- (a) 16 (b) 8 (c) 7 (d) 20 (e) 13

7 Select the graph of the feasible set (F.S.) of the system of linear inequalities given by:

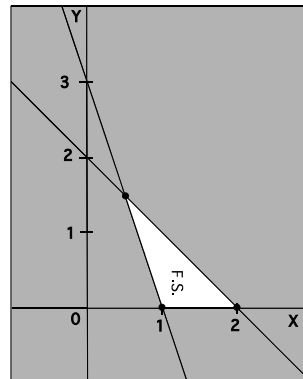
$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ 3x + y &\geq 3 \\ 2x + 2y &\leq 4 \end{aligned}$$



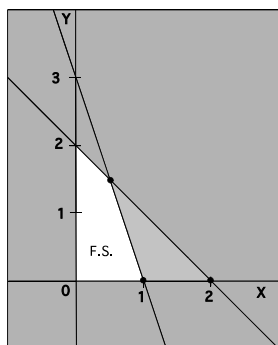
(a)



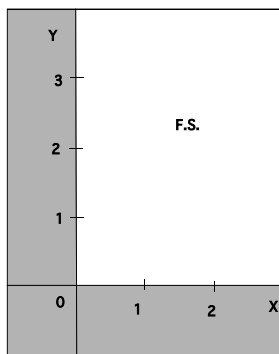
(b)



(c)



(d)



(e)

8 Joe and Mary are about to start a business making surf boards. They will make two models, “The Mulakai” and “The Coolabah”. Joe will shape the boards and Mary will paint them. Mary has 20 hours available to spend on painting the boards each week and Joe has 30 hours per week available to spend painting them. It takes 2 hours to paint “The Mulakai” and 3 hours to paint “The Coolabah”. It takes five hours to shape “The Mulakai” and four hours to shape “The Coolabah”. Let x denote the number of “The Mulakai” made in one week and let y denote the number of “The Coolabah” made in a week. Which of the following sets of inequalities describe the constraints for this problem.

$$\begin{array}{lll}
 (a) \quad \begin{array}{l} 2x + 3y \leq 20 \\ 5x + 4y \leq 30 \\ x \geq 0, \quad y \geq 0 \end{array} & (b) \quad \begin{array}{l} 2x + 5y \leq 30 \\ 3x + 4y \leq 20 \\ x \geq 0, y \geq 0 \end{array} & (c) \quad \begin{array}{l} 2x + 5y \leq 20 \\ 3x + 4y \leq 30 \\ x \geq 0, \quad y \geq 0 \end{array}
 \end{array}$$

$$\begin{array}{ll}
 (d) \quad \begin{array}{l} 2x + 3y \leq 30 \\ 4x + 5y \leq 30 \end{array} & (e) \quad \begin{array}{l} 2x + 3y \geq 0 \\ 5x + 4y \geq 0 \end{array}
 \end{array}$$

9 Find the product

$$\begin{pmatrix} 1 & 0 \\ 2 & 2 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

$$\begin{array}{lllll}
 (a) \quad \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 1 & 2 \end{pmatrix} & (b) \quad \begin{pmatrix} 1 & 2 \\ 2 & 6 \end{pmatrix} & (c) \quad \begin{pmatrix} 1 & 2 & 2 \\ 2 & 6 & 3 \end{pmatrix} & (d) \quad \begin{pmatrix} 1 & 2 \\ 2 & 6 \\ 1 & 5 \end{pmatrix} & (e) \quad \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 3 \end{pmatrix}
 \end{array}$$

10 Let $A = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}$. Find the entry in the second row and first column of A^{-1} .

- (a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) 0 (d) 1 (e) -1.

Partial Credit

11 Abe and Beryl play a game. They throw a dice. If the outcome is a six, Abe gives Beryl \$1 and if the outcome is not a six, Beryl gives Abe \$2.

(a) Let X denote Abes earnings for the game, show the probability distribution for X below.

(b) What are the expected earnings for Abe for this game?

(c) If Abe and Beryl play the game 100 times, roughly how much would Abe be expected to win?

(d) If Abe and Beryl play the game 100 times, roughly how much would Beryl be expected to win?

12 (a) Let z be a standard normal random variable. Draw a rough sketch of the region under the standard normal curve, which corresponds to the probability that z takes a value between -1.5 and 1.5 (that is the region corresponding to $Pr(-1.5 \leq x \leq 1.5)$).

(b) What is the probability that z takes a value between -1.5 and 1.5 , ($Pr(-1.5 \leq z \leq 1.5)$).

(c) Let X denote the scores on the LSAT for a particular year. X had a normal distribution with mean $\mu = 150$ and standard deviation $\sigma = 10$. What percentage of LSAT scores in that year were between 135 and 165?

13 Miss Muffet is in charge of the next field trip for the Notre Dame society of Aracnophobics. She will make survival kits available for the trip.

The Basic survival kit will contain a bottle of anti-venom and a can of spider repellent.

The Deluxe survival kit will have a bottle of anti-venom and two cans of spider repellent and one Tuffet.

Miss Muffet has available 42 bottles of anti-venom, 50 cans of spider repellent and 10 tuffets. She must provide at least 5 Deluxe kits, (since 5 of the members insist on sitting on Tuffets to eat their lunch).

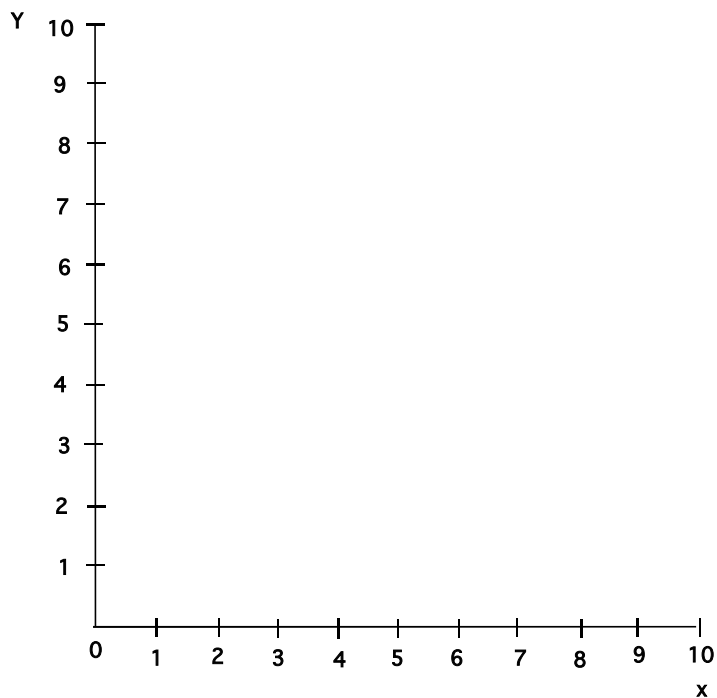
Let x denote the number of Basic kits she makes and let y denote the number of Deluxe kits she makes.

(a) Write the constraints given above in terms of x and y .

(b) The volume of a Basic kit is 25 square inches, the volume of a Deluxe kit is 50 square inches and Miss Muffet wants to minimize the amount of space devoted to survival kits on the coach, what is the objective function describing the amount of space taken up by the survival kits?

14 (a) Graph the feasible set corresponding to the following set of inequalities on the set of axes provided: (Make sure you label the region corresponding to the feasible set clearly)

$$\begin{aligned}x &\geq 0 \\y &\geq 0 \\y + 3x &\geq 9 \\y + x &\geq 5\end{aligned}$$



(b) Find the vertices of the above feasible set.

(c) Find the minimum of the objective function $5x + 2y$ on the above feasible set.

15 (a) Consider the following matrices:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 4 & 7 \\ 5 & 8 & 10 \end{pmatrix} \quad C = \begin{pmatrix} 2 \\ 4 \\ 6 \\ 8 \\ 10 \end{pmatrix} \quad D = (1 \ 3 \ 5 \ 7 \ 9)$$

Which of the following matrices exist?

$$A \cdot B$$

$$B \cdot A$$

$$C + D$$

$$D \cdot C$$

$$A^{-1}.$$

(b) Ruth and Charlie play a game. At each play, Ruth and Charlie simultaneously extend either one or two fingers and call out a number. The player whose call equals the total number of extended fingers wins that many pennies from the opponent. In the event that neither player's call matches the total, no money changes hands. Write down a pay-off matrix for Ruth for this game.

Areas under the Standard Normal Curve

| z | $A(z)$ | z | $A(z)$ | z | $A(z)$ | z | $A(z)$ | z | $A(z)$ |
|-------|--------|-------|--------|------|--------|------|--------|------|--------|
| -3.50 | .0002 | -2.00 | .0228 | -.50 | .3085 | 1.00 | .8413 | 2.50 | .9938 |
| -3.45 | .0003 | -1.95 | .0256 | -.45 | .3264 | 1.05 | .8531 | 2.55 | .9946 |
| -3.40 | .0003 | -1.90 | .0287 | -.40 | .3446 | 1.10 | .8643 | 2.60 | .9953 |
| -3.35 | .0004 | -1.85 | .0322 | -.35 | .3632 | 1.15 | .8749 | 2.65 | .9960 |
| -3.30 | .0005 | -1.80 | .0359 | -.30 | .3821 | 1.20 | .8849 | 2.70 | .9965 |
| -3.25 | .0006 | -1.75 | .0401 | -.25 | .4013 | 1.25 | .8944 | 2.75 | .9970 |
| -3.20 | .0007 | -1.70 | .0446 | -.20 | .4207 | 1.30 | .9032 | 2.80 | .9974 |
| -3.15 | .0008 | -1.65 | .0495 | -.15 | .4404 | 1.35 | .9115 | 2.85 | .9978 |
| -3.10 | .0010 | -1.60 | .0548 | -.10 | .4602 | 1.40 | .9192 | 2.90 | .9981 |
| -3.05 | .0011 | -1.55 | .0606 | -.05 | .4801 | 1.45 | .9265 | 2.95 | .9984 |
| -3.00 | .0013 | -1.50 | .0668 | .00 | .5000 | 1.50 | .9332 | 3.00 | .9987 |
| -2.95 | .0016 | -1.45 | .0735 | .05 | .5199 | 1.55 | .9394 | 3.05 | .9989 |
| -2.90 | .0019 | -1.40 | .0808 | .10 | .5398 | 1.60 | .9452 | 3.10 | .9990 |
| -2.85 | .0022 | -1.35 | .0885 | .15 | .5596 | 1.65 | .9505 | 3.15 | .9992 |
| -2.80 | .0026 | -1.30 | .0968 | .20 | .5793 | 1.70 | .9554 | 3.20 | .9993 |
| -2.75 | .0030 | -1.25 | .1056 | .25 | .5987 | 1.75 | .9599 | 3.25 | .9994 |
| -2.70 | .0035 | -1.20 | .1151 | .30 | .6179 | 1.80 | .9641 | 3.30 | .9995 |
| -2.65 | .0040 | -1.15 | .1251 | .35 | .6368 | 1.85 | .9678 | 3.35 | .9996 |
| -2.60 | .0047 | -1.10 | .1357 | .40 | .6554 | 1.90 | .9713 | 3.40 | .9997 |
| -2.55 | .0054 | -1.05 | .1469 | .45 | .6736 | 1.95 | .9744 | 3.45 | .9997 |
| -2.50 | .0062 | -1.00 | .1587 | .50 | .6915 | 2.00 | .9772 | 3.50 | .9998 |
| -2.45 | .0071 | -.95 | .1711 | .55 | .7088 | 2.05 | .9798 | | |
| -2.40 | .0082 | -.90 | .1841 | .60 | .7257 | 2.10 | .9821 | | |
| -2.35 | .0094 | -.85 | .1977 | .65 | .7422 | 2.15 | .9842 | | |
| -2.30 | .0107 | -.80 | .2119 | .70 | .7580 | 2.20 | .9861 | | |
| -2.25 | .0122 | -.75 | .2266 | .75 | .7734 | 2.25 | .9878 | | |
| -2.20 | .0139 | -.70 | .2420 | .80 | .7881 | 2.30 | .9893 | | |
| -2.15 | .0158 | -.65 | .2578 | .85 | .8023 | 2.35 | .9906 | | |
| -2.10 | .0179 | -.60 | .2743 | .90 | .8159 | 2.40 | .9918 | | |
| -2.05 | .0202 | -.55 | .2912 | .95 | .8289 | 2.45 | .9929 | | |